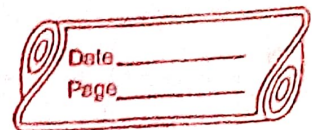


15/02/2024



B.Sc. Part II (Hons)

4th Paper

Linear Diff Equations with constant coefficients

Q Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

Soln For CF, $D^4 + 2D^2 + 1 = 0$
 $\Rightarrow (D^2 + 1)^2 = 0 \Rightarrow D^2 + 1 = 0$

$\Rightarrow D = \underline{+i}, \underline{+i}$

\therefore CF = $(c_1 + c_2 x) e^{ix} + (c_3 + c_4 x) e^{-ix}$

$= (c_1 + c_2 x) (\cos x + i \sin x)$
 $+ (c_3 + c_4 x) (\cos x - i \sin x)$

$= \cos x [(c_1 + c_3) + x(c_2 + c_4)]$

$+ \sin x [(c_1 - c_3) + ix(c_2 - c_4)]$

\therefore CF = $(A + Bx) \cos x + (C + Dx) \sin x$

where $A = c_1 + c_3$, $B = c_2 + c_4$

$C = c_1 - c_3$, $D = i(c_2 - c_4)$

For PI

PI = $\frac{1}{(D^4 + 2D^2 + 1)} x^2 \cos x$

$$\therefore PI = \frac{1}{(D^2+1)^2} x^2 \cos x$$

First we integrate $\cos x$.

Putting $D^2 = -1$ makes denominator zero.

So, we take $\cos x = \text{Real part of } e^{ix}$.

$$\therefore PI = \text{R.P. of } \frac{1}{(D^2+1)^2} x^2 e^{ix}$$

$$\Rightarrow PI = \text{R.P. of } e^{ix} \frac{1}{\left[(D+i)^2+1\right]^2} x^2$$

$$= \text{R.P. of } e^{ix} \frac{1}{(D^2+2Di)^2} x^2$$

$$= \text{R.P. of } e^{ix} \frac{1}{\left[2Di \left(1 + \frac{D}{2i}\right)\right]^2} x^2$$

$$= \text{R.P. of } \frac{-e^{ix}}{4} \frac{1}{D^2} \left(1 + \frac{D}{2i}\right)^{-2} x^2$$

$$= \text{R.P. of } \frac{-e^{ix}}{4} \frac{1}{D^2} \left[1 - \frac{2D}{2i} + \frac{3D^2}{4i^2} - \dots\right] x^2$$

$$= \text{R.P. of } \frac{-e^{ix}}{4} \frac{1}{D^2} \left[x^2 - \frac{2 \times 2x}{2i} - \frac{3 \times 2}{4} - 0\right]$$

$$= \text{R.P. of } \frac{-e^{ix}}{4} \frac{1}{D^2} \left[x^2 + 2ix - \frac{3}{2}\right]$$

$$\therefore \text{PI} = \text{R.P. of } -\frac{e^{ix}}{4} \frac{1}{D} \int (x^2 + 2ix - \frac{3}{2}) dx$$

$$\Rightarrow \text{PI} = \text{R.P. of } -\frac{e^{ix}}{4} \frac{1}{D} \left[\frac{x^3}{3} + 2x^2 - \frac{3}{2}x \right]$$

$$\Rightarrow \text{PI} = \text{R.P. of } -\frac{e^{ix}}{4} \int \left(\frac{x^3}{3} + 2x^2 - \frac{3}{2}x \right) dx$$

$$\Rightarrow \text{PI} = \text{R.P. of } -\frac{e^{ix}}{4} \left[\frac{x^4}{12} + \frac{2x^3}{3} - \frac{3}{4}x^2 \right]$$

$$\Rightarrow \text{PI} = \text{R.P. of } -\frac{1}{4} (\cos x + i \sin x) \left(\frac{x^4}{12} + \frac{2x^3}{3} - \frac{3}{4}x^2 \right)$$

$$\Rightarrow \text{PI} = \text{R.P. of } -\frac{1}{4} \left[\left(\frac{x^4}{12} - \frac{3}{4}x^2 \right) \cos x - \frac{2x^3}{3} \sin x \right]$$

\(\therefore\) complete solution is given by

$$y = \text{CF} + \text{PI}$$

$$\Rightarrow y = (A + Bx) \cos x + (C + Dx) \sin x - \frac{1}{4} \left[\left(\frac{x^4}{12} - \frac{3}{4}x^2 \right) \cos x - \frac{2x^3}{3} \sin x \right]$$